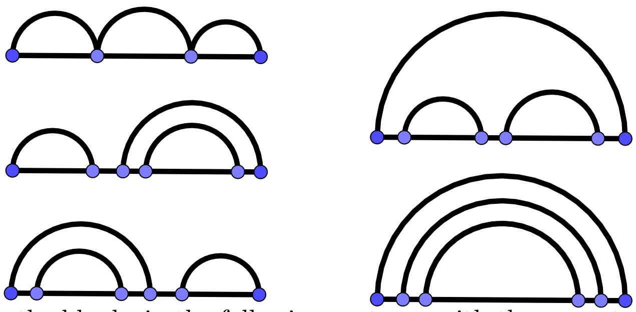
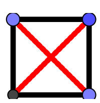
Puzzle Library Problems

1. Consider drawing n semicircles on and above a horizontal line, with no two semicircles intersecting. The results for $n = 3$ are shown below. How many different drawing are there for four semicircles? Draw and label them.



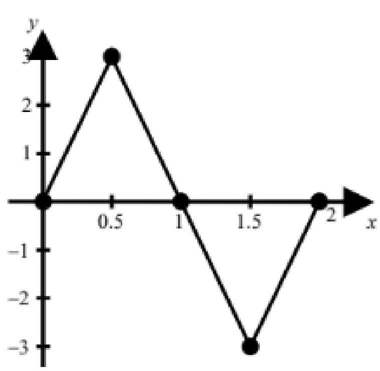
2. Fill in the blanks in the following sentence with the correct numbers: "In this sentence, the number of occurrences of 0 is ___, of 1 is ___, of 2 is ___, of 3 is ___, of 4 is ___, of 5 is ___, of 6 is ___, of 7 is ___, of 8 is ___, of 9 is ___." If it has a solution, find all of them. If not, explain why no solutions exist.

3. How can four distinct point be arranged in the plane so that the six distances between pairs of points take on only two different values? One way is given to the below.



4. The Numerical Dictionary is a special book which contains all the integers written in English in alphabetical order. Determine **(a)** the first entry, **(b)** the last entry, **(c)** the next to last entry

5. The graph of f is shown below. The domain is [0,2]. Find the domain of $f \circ f = f(f(x))$, and sketch it.



6. Clearly sketch the curves and/or regions in the plane defined by the equation $\lfloor x \rfloor^2 + \lfloor y \rfloor^2 = 1$.

7. What is the smallest positive integer not definable in fewer than twelve words. Explain your answer

8. You are in a room that is 5 meters wide, 3 meters high, and 12 meters long. A bug starting in one corner crawls along the inside of the room to the opposite corner. What is the exact length of the shortest path?

9. Give a simple representation of the function:

$$f(x) = \lim_{n\to\infty} \left(\lim_{k\to\infty} (\cos(n!\pi x))^{2k} \right)$$

   We can assume n and k are integers

10. (A quasi-competitive experiment): Choose exactly one of the twelve integers from 1 to 12. The student (or students) that choose the number that occurs least often will receive extra credit.
    There is a second set of data where you know the data from the first round.

11. Let a and b denote constant real numbers and suppose that $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = ax + b$. Conjecture a formula for f^n where $n \in \mathbb{N}$ and n is the number of times f is composed with itself. Write your answer in a closed form (i.e. no summations or ellipses)

12. Draw or describe the graph of the equation $x^2y - y^3 - 5x^2 + 5y^2 = 0$

13. Consider the sequence of non-decreasing positive integers $a_1, a_2, a_3, \ldots$, which has the property that it contains exactly a_k occurrences of k for each positive integer k. List the first 20 terms of this sequence.

14. A group of n students enter a locker room that contains n lockers numbered 1 through n. The first student opens all the lockers. The second student changes the status (from closed to open, or vice versa) of every other locker, starting with the second locker. The third student then changes the status of every third locker, starting at the third locker. In general, for $1 < k \leq n$, the k-th student changes the status of every k-th locker, starting with the k-th locker. After the n-th student has gone through the lockers, which lockers are left open?